The SM Higgs Vacuum Instability, Inflation and the Fate of Our Universe

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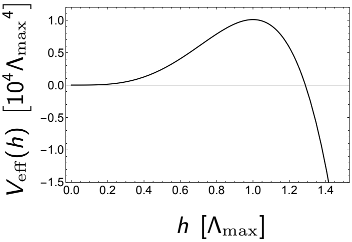
The SM Higgs Potential Instability

In the SM, Higgs quartic coupling $\lambda(\mu)$ runs negative at scales $\mu > \Lambda_I$, producing an unstable potential.



As such, EW vacuum unstable to decay via CdL instanton.

e.g., Sher [Phys.Rept. 179, 273 (1989)], Casas, Espinosa, Quiros [hep-ph/9409458]

We would like this to provide an argument for new physics...but lifetime today exceeds age of universe for measured (m_h, m_t).

e.g., Buttazzo et al. [1307.3536]

BUT...what about during inflation!?

Light scalar fields experience quantum fluctuations $\delta h \sim \frac{H}{2\pi}$ in de Sitter (dS) space due to expansion ($H \equiv$ Hubble during inflation).

- For $H \gtrsim \Lambda_I \sim 10^{10\text{-}13}$GeV $\Rightarrow$ unstable regime sampled during inflation.
  Note: values for Λ_I correspond to 1σ uncertainty on (m_h, m_t).

- Particularly relevant if we observe $r \sim 0.1 \Rightarrow H \sim 10^{14}$GeV.

  Espinosa, Giudice, Riotto [0710.2484], Kobakhidze & Spencer-Smith [1301.2846], Enqvist, Meriniemi, Nurmi [1306.4511], Fairbairn & Hogan [1403.6786]...

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What are the implications of the instability during inflation?

Answering this question requires an understanding of:

1. how Higgs field fluctuations evolve during inflation, and
2. the consequences of field fluctuations for inflation, our universe, etc.

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This is a difficult problem

Really?

Plenty of study of light scalar field fluctuations during inflation.

- *e.g.*, inflaton fluctuations, $\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})$
- "Slow-roll" $\Rightarrow$ fluctuations $\delta\phi(t, \mathbf{x})$ approximately massless.
- Produce local ($\sim H^{-1}$) inhomogeneities in energy density, curvature.

$$\left\langle \delta\phi^2 \right\rangle \Rightarrow \left\langle (\delta\rho/\rho)^2 \right\rangle, \left\langle \mathcal{R}^2 \right\rangle \Rightarrow \left\langle (\delta T_{\rm CMB}/T_{\rm CMB})^2 \right\rangle$$

Similar story should hold for Higgs fluctuations, $\delta h(t, \mathbf{x})$.

- Local variation in Higgs vev, energy density.

  Note: vev in a Hubble patch $\equiv$ sum over superhorizon modes.

So what makes the Higgs special/tricky/especially tricky?

1. The Higgs is not an exclusively "light" field
   - Higgs dynamics governed by *both* dS space and $V(h) \approx \frac{\lambda}{4} h^4$.
   - $V(h)$ dominates for $h \gtrsim h_{\rm classical} \equiv \left(\frac{3}{-2\pi\lambda}\right)^{1/3} H$.
2. The Higgs has non-trivial couplings to other particles
   - V, λ evolve with scale.
   - Gauge invariance issues?

   Andreassen, Frost, Schwartz [1408.0287, 1408.0292]
   Di Luzio, Mihaila [1404.7450]
3. How do we treat large fluctuations?
   - Runaway direction in $V \Rightarrow$ large $\rho_h < 0$.
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Case Study in Confusion: The Hawking-Moss Calculation

Field excited to top of potential barrier with

$$\mathbb{P} = A \exp\left[-\frac{8\pi\Delta V}{3H^4}\right], \qquad \Delta V = V(\Lambda_{\rm max}) - V(0),$$

subsequently rolls down to "true vacuum."

<u>PROS:</u> Gauge invariant, physical.

<u>CONS:</u> Built-in assumption that only care about a patch transitioning to unstable regime. But does unstable $\Rightarrow$ disaster?

- **During inflation:** patches still expand and evolve as long as $\rho_\phi > |\rho_h|$, and causally-disconnected patches should continue to evolve independently.
- **After inflation:** even patches with $h > \Lambda_{\rm max}$ could in principle be stabilized by efficient reheating (which generates $m^2_{h,\rm eff} \sim g^2 T^2$).

Prefactor A for $H^4 \gtrsim \Delta V$?

So, it seems we really care about the **full distribution and evolution** of Higgs vev fluctuations during inflation $\Rightarrow$ require **stochastic approach** to capture dynamics not incorporated by HM.

To tackle this difficult problem, we will break it down into three parts:

1. Develop stochastic approach for toy model in Gaussian approximation
   Study how fluctuations evolve for unstable field
2. Perturbative calculation of correlation function
   Connect stochastic approach to "rigorous" PT, move beyond toy to full SM
3. Fokker-Planck Equation
   Incorporate non-Gaussianity

Hook, JK, Shakya, Zurek [1404.5953]

JK, Yoo, Zurek [1503.05193]

(I) Quartically-Coupled Scalar Evolution in the Hartree-Fock or Gaussian Approximation

Field evolution in dS space

Equation of Motion in dS:

$$\ddot{h} + 3H\dot{h} - \left(\frac{\vec{\nabla}}{a}\right)^2 h + V'(h) = 0$$

- Take $V(h) = \frac{\lambda}{4}h^4$ with $\lambda < 0$,
- Decompose $h(t, \mathbf{x}) = \bar{h}(t) + \delta h(t, \mathbf{x})$ with $\bar{h}(t) = \bar{h}(0) = 0$.

Mode expansion treating field as Gaussian gives

$$\ddot{\delta h}_k + 3H\dot{\delta h}_k + \left\{\left(\frac{k}{a}\right)^2 + 3\lambda\left\langle\delta h^2(t)\right\rangle\right\}\delta h_k = 0$$

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Superhorizon Fluctuation Two-Point Correlation Function

$$\langle \delta h^2(t) \rangle = \int_{k=1/L}^{k=\epsilon a H} \frac{d^3 k}{(2\pi)^3} \, |\delta h_k(t)|^2$$

1. Superhorizon modes only
   - Subhorizon (UV) contributions cancelled by "local" counterterms
   - Dominant effects on superhorizon physics reabsorbed into renormalization—will return to this in (II)
2. IR cutoff
   - Region of space over which $\bar{h}(0) = 0$ is a good approximation, *i.e.*

$$L^{-1} = a_0 H$$

where a_0 is scale factor at onset of inflation.

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For $|\lambda| \left\langle \delta h^2(t) \right\rangle \ll H^2$ and slow-roll,

$$\frac{d}{dt} \left\langle \delta h^2(t) \right\rangle = -\frac{2\lambda}{H} \left\langle \delta h^2(t) \right\rangle^2 + \frac{H^3}{4\pi^2}$$

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Stochastic noise term from time-dependence of horizon crossing.

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SOLUTION:

$$\left\langle \delta h^2(t) \right\rangle = \frac{1}{\sqrt{-2\lambda}} \frac{H^2}{2\pi} \tan\left(\sqrt{-2\lambda} \frac{\mathcal{N}}{2\pi} \right)$$

where $\mathcal{N} = Ht$.

- Unstable potential accelerates growth of fluctuations relative to

$$\left\langle \delta h^2(t) \right\rangle = \frac{H^2}{4\pi^2} \mathcal{N} \qquad (\lambda = 0)$$

- In fact, diverges in finite time! $\mathcal{N}_{\max} = \frac{\pi^2}{\sqrt{-2\lambda}}$

What might the implications of this divergence be?

At $\mathcal{N} \approx \mathcal{N}_{\max}$, Gaussian field distribution becomes very (infinitely) broad. As such, typical vev fluctuations $\sim \sqrt{\langle \delta h^2(t) \rangle}$ in a patch are large.

Consequently, expect a significant portion of patches to be fluctuating to backreacting/crunching regime with $|\rho_h| \sim \rho_\phi$.

- If inflation ends at $\mathcal{N} > \mathcal{N}_{\max}$, resulting universe almost certainly contains a non-negligible proportion of patches that cannot be stabilized by reheating.
    - if these crunch very rapidly, resulting large inhomogeneities and defects likely inconsistent with small perturbations in our Universe, or
    - could nucleate and destroy EW vacuum.
- Moreover, if collapsing patches come to dominate during inflation, entire space may become unstable, see Sekino, Shenker, Susskind [1003.1374].

So, in HF approximation, $\mathcal{N}_{\max}$ is *absolute upper bound* on $\mathcal{N}$.

Comments

1. $\mathcal{N} < \mathcal{N}_{\rm max}$ necessary, but not sufficient...

   Unstable patches present at end of inflation still need to be stabilized.

2. Assumed massless modes and slow-roll...

   Only violated once $\mathcal{N} \sim \mathcal{N}_{\rm max}$.

3. No regulation of (unphysical) divergence

   e.g., should throw away backreacting patches (or those exiting slow-roll)? Fortunately proportion only significant once $\mathcal{N} \sim \mathcal{N}_{\rm max}$.

4. Field treated as Gaussian stochastic variable

   Non-Gaussianity relevant for most unstable (diverging, crunching) patches. Hence, may significantly impact inflationary scenario—see (III).

5. Constant λ

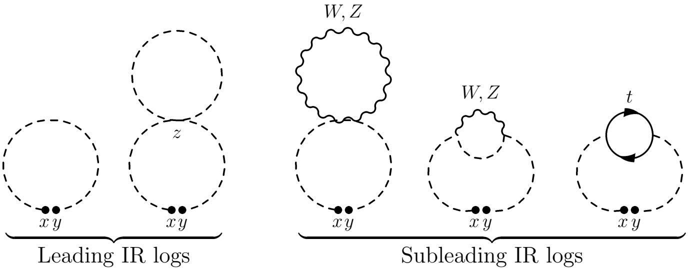
   Whether this makes sense for the Higgs will be addressed in (II).

(II) The Correlation Function in Perturbation Theory

Goals for (II)

1. Understand how a stochastic approach such as HF captures results of a "more traditional" perturbative calculation, and

2. Elucidate how to extend toy model to incorporate rest of SM.



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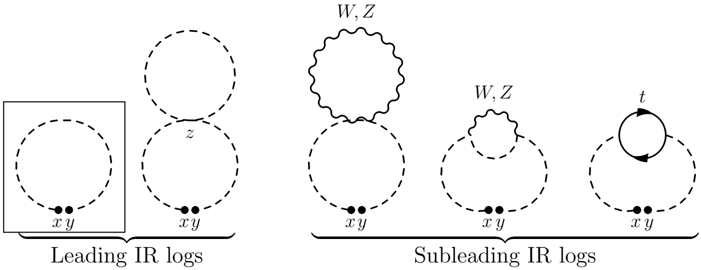
Leading IR logs
Subleading IR logs

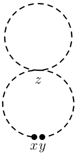


Calculate first diagram, take "coincident limit" $|\mathbf{x}-\mathbf{y}| \approx (aH)^{-1}$...

Leading IR behavior given by

$$\langle \delta h^2(t) \rangle \approx \frac{H^2 \mathcal{N}}{4\pi^2} + \dots$$

One-loop correction with UV (& IR) cutoff



$$\propto 3\lambda \int_{a_0 H}^{a\Lambda} \frac{d^3 k}{(2\pi)^3} |h_k(t_z)|^2 = 3\lambda \left[\frac{\Lambda^2}{8\pi^2} + \frac{H^2}{8\pi^2} \log\left(\frac{a\Lambda}{a_0 H}\right)^2 \right]$$

Two important types of terms

1. UV: Divergences as in Minkowski space (with H relevant energy scale), cancelled by local counterterms

$$\delta m^2(\mu) = -3\lambda(\mu)\frac{\Lambda^2}{8\pi^2}, \qquad 12\delta\xi = -\frac{3\lambda(\mu)}{4\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

   fixing renormalization conditions. $\mu = H$ resums logs.

2. IR: logs contribute to growth of correlator, $\log \frac{a}{a_0} = \mathcal{N}$.

Where do the all-important IR logarithms come from?

Light, minimally-coupled scalar wave functions unsuppressed outside horizon, so (t, k) integrals produce IR logarithms.

Growth of correlator enhanced (for $\lambda < 0$) by scalar loops

$$\left\langle \delta h^2(t) \right\rangle \approx \frac{H^2 \mathcal{N}}{4\pi^2} - \frac{\lambda H^2 \mathcal{N}^3}{24\pi^2} + \ldots$$

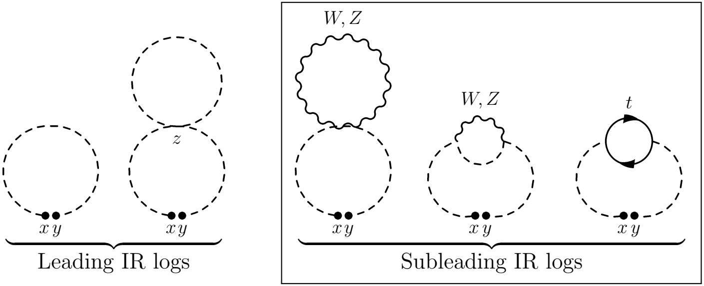
PT breaks down for $\mathcal{N} > \pi \sqrt{\frac{6}{|\lambda|}} \gtrsim \mathcal{N}_{\rm max}$! Moreover, for $\sqrt{-\lambda}\mathcal{N} \ll 1$,

$$\left\langle \delta h^2(t) \right\rangle_{\rm HF} \approx \frac{H^2 \mathcal{N}}{4\pi^2} - \frac{\lambda H^2 \mathcal{N}^3}{24\pi^2} + \ldots$$

So *stochastic approach resums leading IR logarithms.* ✓

See, *e.g.*, Tsamis, Woodard [gr-qc/0505115], Garbrecht, Rigopoulos, Zhu [1310.0367]

So what about the rest of the Standard Model?



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Transverse gauge bosons, fermions damped outside horizon $\Rightarrow$ do not *directly* contribute to leading IR logarithms...

- Leading contributions calculated including only scalar loops

...but high-energy subhorizon modes do see local (flat) spacetime!

- Generate usual logarithms of form $\log\left(\frac{\mu^2}{H^2}\right)$.

e.g., V_{eff} in dS space: Herranen, Markkanen, Nurmi, Rajantie [1407.3141]

- Choose $\mu \approx H$ to control PT, resum large logarithms.

So $\lambda =$ *RG-improved SM quartic* evaluated at $\mu = H$, $\lambda(\mu = H)$. ✓

- Gauge-invariant, physical.

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(III) The Fokker-Planck Approach

The Fokker-Planck Equation

Calculates $P(\delta h, t) \equiv$ probability to observe δh in a patch at time t

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \delta h}\left[\frac{V'(\delta h)}{3H} P + \frac{H^3}{8\pi^2}\frac{\partial P}{\partial \delta h}\right]$$

Related to correlation functions via

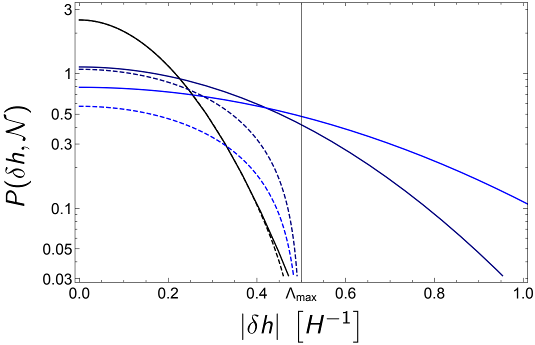
$$\langle \delta h^n(t) \rangle = \int d\delta h \, (\delta h)^n P(\delta h, t)$$

Advantage relative to HF? Incorporates non-Gaussianity.

The Fokker-Planck approach has been used to study the Higgs previously by Espinosa, Giudice and Riotto [0710.2484], but

- employed running coupling $\lambda(\mu = h)$, and
- inappropriately truncated FP solution, artificially suppressing P.

e.g., for $H = 2\Lambda_{\max}$ and $\lambda = -0.01$



Dotted assumes

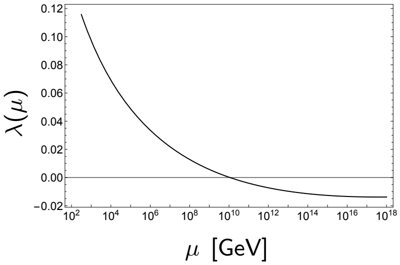
$P(|\delta h| \geq \Lambda_{\max}, \mathcal{N}) = 0$

For $\mathcal{N} = 1, 5, 10$.

So what have we learned from (I) and (II)?

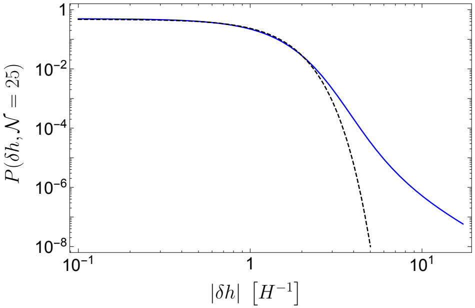
1. Stochastic approach using $V(h) = \frac{\lambda}{4} h^4$ with $\lambda = \lambda(H)$ should unambiguously capture leading IR divergent behavior for SM Higgs.

   Typically, $-0.015 \lesssim \lambda(H) \lesssim -0.005$ in the SM. *e.g.*, for best-fit (m_h, m_t)



2. Unreasonable to truncate FP solution at $|\delta h| = \Lambda_{\max}$ as these patches can still evolve during inflation.

Unstable patches with $\delta h \gtrsim \delta h_{\text{classical}} \equiv \left(\frac{3}{-2\pi\lambda}\right)^{1/3} H$ quickly roll away



Taking $\lambda(H) = -0.01$ ($\Rightarrow \delta h_{\text{cl}} \approx 4H$):

Fokker-Planck

Hartree-Fock with

$$\left\langle \delta h^2(t) \right\rangle = \frac{1}{\sqrt{-2\lambda}} \frac{H^2}{2\pi} \tan\left(\sqrt{-2\lambda}\frac{\mathcal{N}}{2\pi}\right)$$

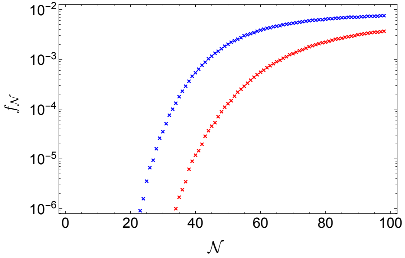
These patches give large contributions to correlation functions

Divergence of correlators $\not\Rightarrow$ significant proportion of space becoming unstable.

Once a patch exits slow-roll for $|\delta h| \gtrsim \delta h_c \equiv \left(\frac{3}{-\lambda}\right)^{1/2} H$, the vev diverges rapidly and the patch appears to evolve to a singularity within one e-fold.

Any such patches present at the end of inflation likely cannot be stabilized even by efficient reheating...but probably crunch before nucleating.

Consider proportion surviving space that is becoming unstable at $\mathcal{N}$



$$f_{\mathcal{N}} \equiv \frac{\int_{-\delta h_c}^{\delta h_c} d\delta h \left\{P(\delta h, \mathcal{N}) - P(\delta h, \mathcal{N}-1)\right\}}{\int_{-\delta h_c}^{\delta h_c} d\delta h \, P(\delta h, \mathcal{N}-1)}.$$

$\lambda(H) = -0.010$

$\lambda(H) = -0.005$

Approach "steady state" where small proportion of space is "sloughed off."

Majority of space never unstable, but defects generated at end of inflation.

The Fate of Our Universe

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But what is the correct information to extract from this formalism?

All depends on how the various patches behave!

Ultimately, we are left with a distribution of patches that are either

- stable ($|\delta h| < \Lambda_{\max}$),
- unstable but still inflaton-dominated ($\Lambda_{\max} \leq |\delta h| \lesssim \sqrt{HM_P}$), or
- rapidly diverging, backreacting and (probably) crunching.

So what is the impact of the various patches on the resulting universe?

Unfortunately, that appears to be a very complicated question.

- If "true vacuum" patches not stabilized during reheating and nucleate before crunching, a single one could be disastrous for our universe. $H \ll \Lambda_{\rm max}$ or NP!

  Kobakhidze & Spencer-Smith [1301.2846], Enqvist, Meriniemi, Nurmi [1306.4511], Fairbairn & Hogan [1403.6786] Hook, JK, Shakya, Zurek [1404.5953]

- If true vacuum patches crunch "benignly," can inflate to replace lost patches.

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  notably, if $f_{\mathcal{N}} \sim \mathcal{O}(0.5)$ required for whole space to crunch, never abort inflation.

- But even if only most unstable patches crunch, $\exists$ a minimum level of defects formed at end of inflation. Could potentially imply a bound on high-scale inflation.

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  e.g., if defects are light, rapidly-evaporating PBHs $\Rightarrow M_P$ relics, $f_{\mathcal{N}}^{\rm crit} \ll 10^{-10}$ to avoid overclosure. If no relics, PBHs simply contribute to reheating.

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- But even if only most unstable patches crunch, $\exists$ a minimum level of defects formed at end of inflation. Could potentially imply a bound on high-scale inflation.

  JK, Yoo, Zurek [1503.05193]

  e.g., if defects are light, rapidly-evaporating PBHs $\Rightarrow$ M_P relics, $f_{\mathcal{N}}^{\rm crit} \ll 10^{-10}$ to avoid overclosure. If no relics, PBHs simply contribute to reheating.

Unfortunately, that appears to be a very complicated question.

- If "true vacuum" patches not stabilized during reheating and nucleate before crunching, a single one could be disastrous for our universe. $H \ll \Lambda_{\rm max}$ or NP!

  Kobakhidze & Spencer-Smith [1301.2846], Enqvist, Meriniemi, Nurmi [1306.4511], Fairbairn & Hogan [1403.6786] Hook, JK, Shakya, Zurek [1404.5953]

- If true vacuum patches crunch "benignly," can inflate to replace lost patches.

  Espinosa, Giudice, Riotto [0710.2484], Hook, JK, Shakya, Zurek [1404.5953]
  JK, Yoo, Zurek [1503.05193]

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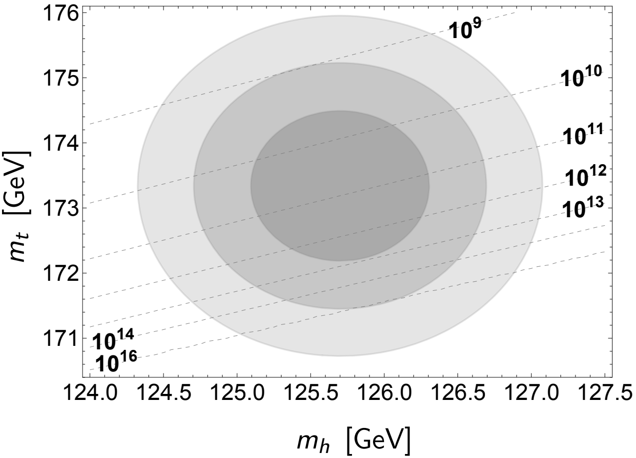
So what can we say about high-scale inflation?

1. That patches would fluctuate to the unstable regime *during* inflation does not appear to preclude high-scale inflation.
   - $\rho_\phi > |\rho_h|$ patches can still inflate...could be stabilized during RH.
   - causally-disconnected patches should evolve independently, permitting EW vacuum to persist in presence of true vacuum patches.
   - $f_{\mathcal{N}} \ll 1 \Rightarrow$ inflation assumedly not aborted.
2. HOWEVER, post-inflationary epoch may need to exhibit certain features to be consistent with instability.
   - *e.g.*, assumedly must at least stabilize true vacuum patches that do not crunch rapidly so they do not nucleate and destroy EW vacuum.
   - rapidly crunching patches likely generate defects of sort usually diluted by inflation—cosmological bounds may be relevant.

Thank you!



Λ_I in GeV. Contours show $(1,2,3)\sigma \Rightarrow 10^9$ GeV $\lesssim \Lambda_I \lesssim 10^{16}$ GeV at 2σ.

Would-be GBs?

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \bar{h} + \delta h + i\chi_3 \end{pmatrix}$$

χ_i eaten for $\langle \mathcal{H}^\dagger \mathcal{H} \rangle \neq 0$, but light for $g^2 \langle \mathcal{H}^\dagger \mathcal{H} \rangle \lesssim H^2$.

If remain light,

$$\langle \chi_i^2 \rangle \approx \langle \delta h^2 \rangle \qquad \Rightarrow \qquad \lambda \to 2\lambda,$$

But, this is violated before PT breaks down [*i.e.*, contributions at $\mathcal{O}(\lambda g^2)$].

"Actual" SM limit in Gaussian approximation:

$$\frac{\pi^2}{2\sqrt{-\lambda(H)}} \lesssim \mathcal{N}_{\max} \lesssim \frac{\pi^2}{\sqrt{-2\lambda(H)}}$$